On-Line PID Controller Tuning from a Single, Closed-Loop Test

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Proportional, integral, and derivative (PID) controllers are still used widely in industry because of their simplicity, robustness and successful practical applications. For such controllers, Yuwana and Seborg (1982) have presented an excellent on-line tuning method which avoids significant disadvantages of the two most popular methods: the time-consuming trial-and-error test requirement for the continuous cycling method (Ziegler and Nichols, 1942) and the open-loop test requirement for the process reaction curve method (Cohen and Coon, 1953). In the Yuwana-Seborg method, a second-order, time-delay transfer function of the closed-loop system model is first estimated from response data of a small step change in controller set point: then a simple process model, a first-order, time-delay transfer function, is back calculated, which approximates the closed-loop model when the loop is connected. The controller settings are then calculated from the process model as in the process reaction curve method. It provides excellent initial settings for the PID controller from a single closed-loop test, without any computational difficulty. But their method, as stated in their paper, fails to provide good settings for a process with very large time-delay, due to their use of the Pade approximation in calculation of parameters of a process model.

In this note, the calculation of parameters of a process model is modified to find better controller settings for a process with large time delay. A model reduction technique (Ouyang et al., 1987) which does not resort to the Pade approximation is utilized. The modified method requires solving a nonlinear equation iteratively, but it is expected to provide a more accurate process model with better control settings. Considering the difficulty of controlling a process with large time delay, this modification would be important in practice and would make the Yuwana-Seborg method be more useful.

Theoretical Development

Consider a simple closed-loop control system as shown in Figure 1. It is assumed that $G_c(s) = K_c$ and the feedback gain K_c is large enough so that the closed-loop system is underdamped. A typical response to step change in the set point is also shown in Figure 1. A closed-loop transfer function which is a second-order, time-delay transfer function and approximates the response of Figure 1, can be obtained easily as (Coughanowr and Koppel, 1965; Yuwana and Seborg, 1982)

$$\frac{C(s)}{R(s)} \simeq \frac{K(qs+1)e^{-ds}}{\tau^2 s^2 + 2\zeta\tau s + 1} \tag{1}$$

where

$$K = c_\infty / A \tag{2}$$

$$\zeta = -\ln\left(\frac{c_\infty - c_{m1}}{c_{p1} - c_\infty}\right) \Bigg/ \sqrt{\pi^2 + \left[\ln\left(\frac{c_\infty - c_{m1}}{c_{p1} - c_\infty}\right)\right]^2} \tag{3}$$

$$\tau = \Delta t \sqrt{1 - \zeta^2}/\pi \tag{4}$$

The measurements, c_{p1}, c_{m1}, c_∞, Δt, are given in Figure 1, and A is the magnitude of step change in the set point.

We choose a first-order, time-delay transfer function as a process model:

$$G_m(s) = \frac{K_m e^{-d_m s}}{\tau_m s + 1} \tag{5}$$

Then we have

$$\frac{C(s)}{R(s)} \simeq \frac{K_c K_m e^{-d_m s}}{1 + \tau_m s + K_c K_m e^{-d_m s}} \tag{6}$$

The parameters of the process model, K_m, τ_m, d_m, with which Eq. 6 will approximate Eq. 1 are sought. For this, Yuwana and Seborg (1982) adopted a Pade approximation of the time-delay term in the denominator of Eq. 6. But here, instead of approximation of the time-delay term, we make the dominant poles of the transfer function of Eq. 6 equal to the poles of Eq. 1. That is,

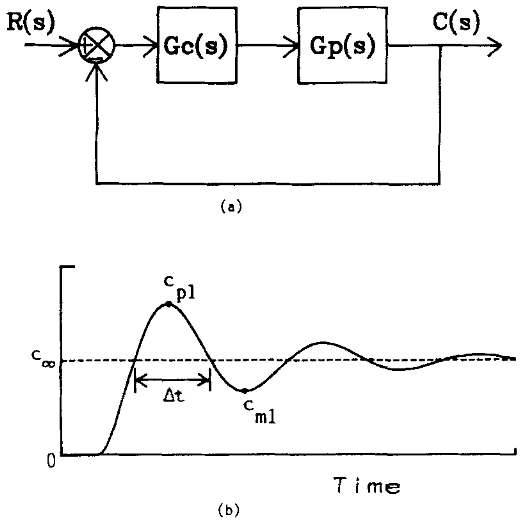

Figure 1. a) Block diagram of a simple closed-loop feedback system.
b) Typical response to a step change in controller set point.

since the poles of Eq. 1 are

$$-\frac{\zeta}{\tau}+\frac{\sqrt{1-\zeta^2}}{\tau}i \quad \text{and} \quad -\frac{\zeta}{\tau}-\frac{\sqrt{1-\zeta^2}}{\tau}i, \quad i=\sqrt{-1},$$

we have

$$1+\tau_m\left(-\frac{\zeta}{\tau}+\frac{\sqrt{1-\zeta^2}}{\tau}i\right)+K_cK_me^{-d_m(-\zeta/\tau+\sqrt{1-\zeta^2}/\tau i)}=0 \quad (7)$$

$$1+\tau_m\left(-\frac{\zeta}{\tau}-\frac{\sqrt{1-\zeta^2}}{\tau}i\right)+K_cK_me^{-d_m(-\zeta/\tau-\sqrt{1-\zeta^2}/\tau i)}=0 \quad (8)$$

By comparing the steady-state gains of Eqs. 1 and 6 we obtain

$$K_m=\frac{c_\infty}{K_c(A-c_\infty)} \quad (9)$$

And from Eqs. 7 and 8, we have

$$e^{-\alpha d_m}-\frac{K_cK_m}{\beta}\left[\alpha\sin(\beta d_m)-\beta\cos(\beta d_m)\right]=0 \quad (10)$$

$$\tau_m=\frac{1}{\alpha}\left[1+K_cK_me^{\alpha d_m}\cos(\beta d_m)\right] \quad (11)$$

where $\alpha=\zeta/\tau$ and $\beta=\sqrt{1-\zeta^2}/\tau$

The first positive solution of Eq. 10 is used as d_m. Many numerical methods to find the solution are available (Rice, 1983). Here, a fixed point iteration method is used:

$$d_{m,k+1}=\frac{1}{\beta}\left[\nu+\tan^{-1}\left(\frac{\beta e^{-\alpha d_{m,k}}}{K_cK_m\sqrt{\alpha^2+\beta^2}\cos(\beta d_{m,k}-\nu)}\right)\right]$$

$$d_{m,0}=(\nu+\pi/4)/\beta \quad (12)$$

where $\nu=\tan^{-1}(\beta/\alpha)$ and k is the iteration counter. About five iterations were sufficient to find three significant digits for most problems.

The process parameters, K_m, τ_m, d_m, calculated from Eqs. 9, 10 and 11 are then used to determine controller settings through tuning relations such as the Ziegler-Nichols rule (Coughanowr and Koppel, 1965).

Simulation Study

Two examples are presented. The first example shows how to estimate the process parameters accurately when the process is of the model form. The second example shows the performance when applied to a process with large time-delay for which the Yuwana-Seborg method is inadequate.

Example 1:

This example considers the process of the form

$$G_p(s)=\frac{e^{-d_ps}}{s+1}$$

Table 1 compares estimates of parameters obtained for runs of two different K_c's and three different d_p's. It shows that the modified method provides estimates of parameters well within a 5% error throughout d_p and K_c. The Yuwana-Seborg method also provides good estimates for small d_p's, but estimates become worse as d_p grows.

Example 2:

This example is chosen from Yuwana and Seborg (1982) where

$$G_p(s)=\frac{e^{-3s}}{(s+1)^2(2s+1)}$$

Table 2 shows estimates of parameters and the corresponding PID controller settings including those shown in Yuwana and Seborg (1982). Figures 2 and 3 compare performances of the modified method (values for the case $K_c=1$ are used). We can see that the modified method provides a process model which traces the process more accurately and provides better settings of the PID controller, somewhat more conservative than the Ziegler-Nichols settings.

Notation

A = magnitude of the step change in controller set point
c, $C(s)$ = controlled variable and its Laplace transform

Table 1. Estimates of Parameters for Example 1

Process				Yuwana-Seborg			Proposed		
K_p	τ_p	d_p	K_c	K_m	τ_m	d_m	K_m	τ_m	d_m
1	1	0.5	1.5	1	1.02	0.507	1	1.01	0.516
			2	1	1.03	0.554	1	0.978	0.520
1	1	1.0	1	1	1.15	0.988	1	0.994	1.01
			1.5	1	1.27	1.14	1	1.03	1.05
1	1	2.0	0.5	1	1.46	1.67	1	0.977	2.07
			1	1	1.59	2.09	1	1.04	2.03

Table 2. Estimates of Parametes and PID Controller Settings for Example 2

Method	Model K_c	Model K_m	Model τ_m	Model d_m	PID Controller Settings* K_c	PID Controller Settings* T_i	PID Controller Settings* T_d
Yuwana-Seborg	1	1	3.92	4.69	1.21	7.04	1.76
	0.25	1	3.57	3.19	1.47	5.03	1.26
Proposed	1	1	2.78	4.74	0.986	6.70	1.67
	0.25	1	2.60	4.87	0.941	6.76	1.69
Ziegler-Nichols					1.04	6.45	1.61

*Ziegler-Nichols rule is used.

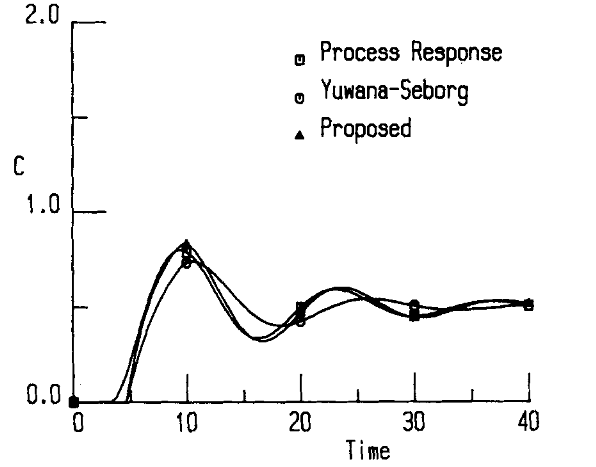

Figure 2. Closed-loop response of P controller, $K_c = 1$, for the process and models of Ex. 2.

c_{m1}, c_{p1}, c_∞ = first minimum, first peak and steady-state value of c
d, d_m, d_p = time delay, model time delay and process time delay
$G_c(s), G_m(s), G_p(s)$ = controller, model and process transfer functions
K, K_c, K_m = closed-loop gain, controller gain and model gain
$R(s)$ = Laplace transform of set point
s = Laplace transform variable

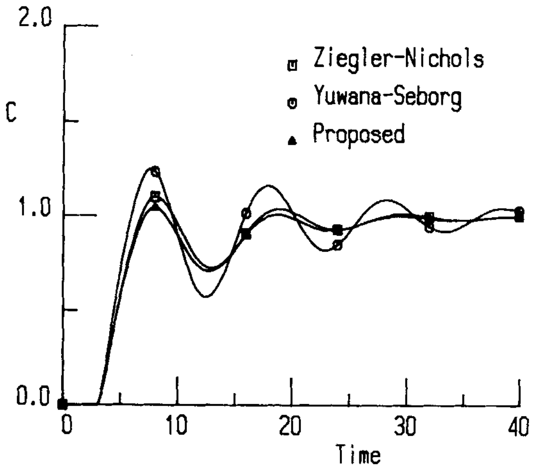

Figure 3. Closed-loop response of PID controllers for the process of Ex. 2.

$t, \Delta t$ = time, half-period of oscillation
T_d, T_i = derivative time and integral time of PID controller
α, β, ν = constants
ζ = damping coefficient for second order model
τ, τ_m = time constant and model time constant

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